The University of Georgia
Department of Mathematics and Science Education
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Probability Instructional Unit

Mathematics 1, Unit 4
by
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Day #1

MM1D1a: Apply the addition and multiplication principles of counting.

Essential Question:
What is the difference between the addition and multiplication principles of counting?

Activities:
To begin this lesson, start with the GSP sketch for MM1D1a. Have the students write down the example from the sketch for their notes. Let the students answer the prompts in the sketch. When the teacher is finished going through the example, use the example to define the Counting Principles of Addition and Multiplication.

> **Counting Principle of Addition**: Let A and B be two events with no common outcomes. The number of times A **or** B will occur is n(A) + n(B) where n(A) is the number of outcomes for event A and n(B) is the number of outcomes for event B.
>
> **Counting Principle of Multiplication**: If there are *m* ways to make a first selection and *n* ways to make a second selection, then there are *m* X *n* ways to make the two selections.

Emphasize that **OR** implies addition and **AND** implies multiplication.

After going over the definitions, the students will work individually on the following two problems:

> **Problem #1:**
> If I have 4 shirts, 5 pairs of pants, and 6 pairs of shoes to choose from, how many possible outfits of 1 shirt, 1 pair of pants, and 1 pair of shoes can I choose from?
>
> Now, I know that I am going to wear my khaki pants, so how many possible outfits do I have to choose from?
>
> **Problem #2:**
> There is a 4 symbol code that can use any letter and/or any number. If any number or letter can repeat itself any number of times in the code, how many possible codes are there to choose from? (Hint: You might need to consider several cases of different arrangements of numbers and letters)

For the remainder of the period, the students will be able to work on their homework. See worksheet in Probability IU Supplement Folder.

Day #2

MM1D1b: Calculate and use simple permutations and combinations.

Essential Question:
What is the difference between permutations and combinations?

Activities:
Begin the lesson with the GSP sketch for MM1D1b (there are 3 different sketches). The students will need to take notes from each sketch. The first sketch introduces the notion of factorials with an example using books.

The second sketch introduces permutations with another example, explains why it's a permutation, and then shows the formula for permutations. Here, the teacher will need to reinforce what is on the sketch and work through the formula with the students. The teacher might want to do another example so students can have more practice with the formula (Example: ${}_8P_5$).

The third and final sketch for this Standard introduces combinations with another interactive example. Again, the teacher will need to show the students how to work with the formula and give another example (Example: ${}_8C_5$).

For the remainder of the period, the students will work on a permutations and combinations worksheet, either individually or in a small group. See worksheet in the Probability IU Supplement Folder.

Day #3

MM1D2a: Find the probabilities of mutually exclusive events.

Essential Question:
How do we find the probability of mutually exclusive events?

Activities:
Before introducing new material, the teacher and students will have a discussion about probability, defining probability and doing a few examples.

Probability: the likelihood of an event occurring.

For some event E, the probability of E occurring is denoted as P(E).
P(E) = The number of favorable outcomes ÷ The total number of outcomes

How can we represent probability? Fractions - between 0 and 1
Decimals - between 0 and 1
Percents - between 0% and 100%

Examples: If I toss a coin and event A is landing on tails, what is P(A)?

If I roll a six-sided number cube (a die) and event B is getting a number less than 3, what is P(B)?

After discussing probability, show the students the GSP sketch to introduce mutually exclusive and overlapping events. The sketch provides a definition and formula for both types of events. Use the Venn diagram in the sketch to show the students where the formulas come from. The students should add the definitions and formulas to their notes along with the following examples:

Let event A be rolling a die and event B be flipping coin. These two events are *mutually exclusive* because they share no common outcomes.

Let event C be rolling a die and landing on 2 and event D be rolling a die and landing on an even number. These two events are *overlapping* because they share a common outcome, 2.

Once the students understand the difference between mutually exclusive events and overlapping events, the teacher will do examples with the class finding the probability of mutually exclusive and overlapping events. Emphasize that when finding the probability of mutually exclusive and overlapping events, we are finding the probability of one event **OR** another event occurring (**OR** implies addition).

Examples: Find P(A or B).

1. Event A: Rolling a die and landing on 3
   Event B: Rolling a die and landing on 6.

2. Event A: Rolling a die and landing on a 2.
   Event B: Rolling a die and landing on an even number

3. Event A: Drawing the Ace of spades from a standard deck of 52 cards.
   Event B: Drawing the King of diamonds from a standard deck of 52 cards.

4. Event A: Drawing a King from a standard deck of 52 cards.
   Event B: Drawing a heart from a standard deck of 52 cards.

For the remainder of the period, the students will be able to work on their homework: Probability IU Supplement Folder: Practice A, #1-6. Practice B, #1-6

Day #4

MM1D2b: Find the probabilities of dependent events.

Essential Question:
How do we find the probability of dependent events?

Activities:
Start the lesson with the example from the GSP sketch. After getting through the example, have a discussion with the students about independent and dependent events and replacing and not replacing marbles in the bag. After getting the input from the students, define independent and dependent events and provide examples, maybe even using some of the ideas from their discussion.

Independent Events: Two or more events that DO NOT affect each other's outcomes.
Example: Rolling a die twice.

Dependent Events: Two or more events that DO affect each other's outcomes.
Example: Dealing two cards (without replacing the first card dealt)

Once the students understand the difference between independent and dependent events, the teacher will do examples with the class finding the probability of independent and dependent events. Emphasize that when finding the probability of independent and dependent events, we are finding the probability of one event **AND** another event occurring (**AND** implies multiplication).

Examples:

1. When rolling a die twice, what is the probability of landing on an even number first and landing on 2 second?

2. If you are dealt 2 cards from a standard deck of 52 cards, and they are **not** replaced after each deal, what is the probability of getting the King of hearts **and** then the Queen of hearts?

The students will work on the Marbles and Cards Learning Task for the remainder of the period and complete it for homework. See task in Probability IU Supplement Folder.

Day #5

MM1D2c: Calculate conditional probabilities.

Essential Question:
What are conditional probabilities and how do we find them?

Activities:
For this lesson, it will be best to start with the GSP sketch for conditional probabilities. There are two sketches with examples that involve eliminating certain outcomes based on a given condition. The students should add the examples to their notes. After going through the two examples, define conditional probability.

> **Conditional Probability**: The probability that one event has occurred ***given*** that another event has occurred.

For the remainder of the period, the students work on a quiz:

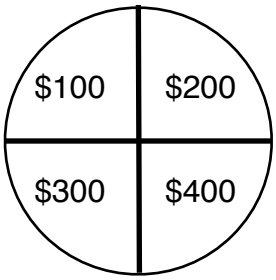
Spinner Learning Tasks that incorporate some basic probabilities and conditional probabilities. See Quiz in the Probability IU Supplement Folder.

Day #6

MM1D2d: Use expected value to predict outcomes.

Essential Question:
What is expected value and how do we use it to predict outcomes?

Activities:
To start this lesson, start with a spinner divided into four equal sections, $100, $200, $300, and $400. Ask the students on **average**, how much would you expect to get when you spin the spinner once?



Hopefully, the students will find the average of the amounts on the spinner. When they do this, they should get (100+200+300+400)/4 = $1000/4 = $250.

Use the expression on the left of the equation above to expand so that students can see that you are multiplying each value and its probability and adding them all to get the expected value. In other words,

(100+200+300+400)/4 = 100/4 + 200/4 + 300/4 + 400/4 = 1/4(100) + 1/4(200) + 1/4(300) + 1/4(400).

After showing the students this, define expected value and include the formula.

> **Expected Value**: The average value of an experiment or situation that is calculated by taking each outcome and multiplying its value by the probability of that outcome, then adding all the products up. In other words, for values x_i and probabilities p_i,
> Expected Value = $p_1x_1 + p_2x_2 + \ldots + p_nx_n$, where $p_1 + p_2 + \ldots + p_n = 1$.

Some students might be be confused why the expected value is $250 when there is not space on the spinner that is $250. Reinforce the idea that the value is just what is expected, not guaranteed.

To show the students that expected value is like an average, open the Expected Value Fathom file to do a simulation with a spinner with eight congruent sections. When the students are done with the simulations, have them calculate the actual expected value.

For the rest of the class, the students will work on a worksheet calculating expecting values for various experiments. See worksheet in the Probability IU Supplement Folder.

Day #7, #8 (Activity might take longer than one day)

MM1D3a: Compare summary statistics (mean, median, quartiles, and interquartile range) from one sample data distribution to another sample data distribution in describing center and variability of the data distributions.

Essential Question:
What are summary statistics, how do we find them, and what do they tell us?

Activities:
For this lesson, the students will start with the definitions of the summary statistics. The students may look them up in the textbook, or the teacher may use these:

Mean: The sum of the numbers in a set of data divided by the number of pieces of data.
Median: The number in the middle of a set of data when the data are arranged in order.
Mode: The number that occurs most frequently in a set of numbers.
Range: The difference between the largest and smallest values in a numerical data set.
Lower Quartile (Q1): After dividing a set of data at the median, the lower quartile value is the median of the lower half of the data.
Upper Quartile (Q3): After dividing a set of data at the median, the upper quartile value is the median of the upper half of the data.
Interquartile Range (IQR): The difference between the first and third quartiles.
Central Tendency: A measure of the "middle" or "expected" value of the data set
Measures of Central Tendency: Mean, Median, and Mode
Dispersion/Variability/Spread: The degree to which differences exist among a set of scores.
Measures of Dispersion/Variability/Spread: Range and Interquartile Range

After the students add the definitions to their notes, do an example with the students to practice finding the summary statistics.

Example: The following numbers indicate the number of points George scored in 10 basketball games. Find the summary statistics of the data.

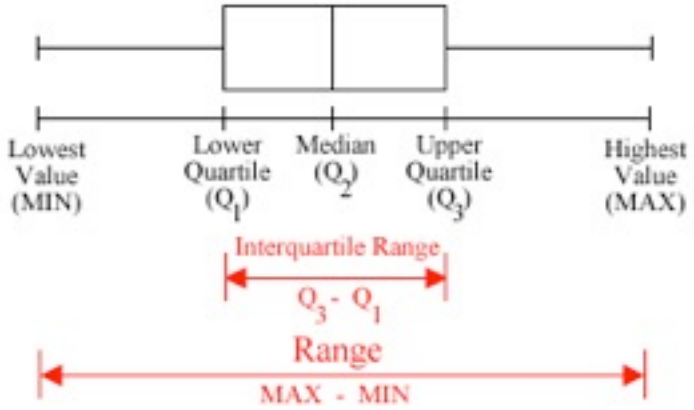
19, 8, 12, 11, 5, 21, 15, 14, 25, 17

After the students complete the example above, open the Comparing Statistics Fathom file. With this file, the teacher will use Fathom to find the summary statistics of two sets of data of distances. After the statistics for both sets of data are found, the students will compare the statistics of the two sets of data. The teacher will also show the students a box and whisker plot of the data.

The students will return to the set of data from the first example used in class and will make a box and whisker plot using the data. They will need to add the following instructions to their notes:

To Make Box and Whisker Plots:
1. Start with a number line that is representative of the set of data.
2. Draw a line segment at the value of the **median** above the number line.
3. Draw a line segment at the value of the **lower quartile** above the number line.
4. Draw a line segment at the value of the **upper quartile** above the number line.
5. Draw a point at the minimum data value (above the number line).
6. Draw a point at the maximum data value (above the number line).

Connect the line segments to make a box and draw segments from the lower quartile to the minimum point and from the upper quartile to the maximum point. See example below.



The students will be given a worksheet with two more sets of data and will have to compare their summary statistics. See worksheet in the Probability IU Supplement Folder.

Day #9

MM1D3b: Compare the averages of the summary statistics from a large number of samples to the corresponding population parameters.
MM1D3c: Understand that a random sample is used to improve the chance of selecting a representative sample.

Essential Question:
What is the difference between a sample and a population and why do we compare their statistics?

In what kind of situation would a random sample be more representative of a population than a biased sample?

Activities:
This lesson will start with a discussion about populations and samples. In their notes, the students will need to write the following definitions:

> **Population**: Represents all possible measurements or outcomes that are of interest to us in a particular study.
>
> **Sample**: Refers to a portion of the population that is representative of the population from which it was selected.
>
> *Example*: If the population is all Apalachee High School students, then a sample could be 100 Apalachee High School students.
>
> Explain to the students that are several types of samples, but only two are of interest to us:
>
> **Representative Sample**: A collection of objects that includes all parts of the population of the objects in their proper proportions. *Random* sampling increases the chance of having more representative samples.
>
> **Biased Sample**: A sample that does not contain units in the same proportion as the population of interest
>
> *Example*: If conducting a survey on high school students that enjoy classical music, a biased sample would be asking all band students. A more representative sample would be to ask 100 high school students chosen at random.

After taking a few notes, the students will compare the summary statistics of a sample to those of a population using the Sample v. Population Fathom file. They will compare the number of car accidents in one state and compare it to the number of accidents in the entire country. The teacher will have to show the students how to find the summary statistics using Fathom.

Day #10

MM1D4: Students will explore variability of data by determining the mean absolute deviation (the average of the absolute values of the deviations).

Essential Question:
What is mean absolute deviation, how do we find it, and what does it tell us?

Activities:
The class will start with the students taking notes on how to find Mean Absolute Deviation:

> Step 1: Calculate the mean of a data set.
> Step 2: Subtract the mean from each value in the data set. These are called the *deviations*.
> Step 3: Take the absolute value of each deviation.
> Step 4: Calculate the mean of each absolute deviation.
>
> The Mean Absolute Deviation is also a measure of dispersion/variability/spread of a set of data.

Give the students an example so that they can practice finding the Mean Absolute Deviation.

> *Example*: The following numbers indicate the number of points George scored in 10 basketball games. Find the Mean Absolute Deviation of the data.
>
> 19, 8, 12, 11, 5, 21, 15, 14, 25, 17

Next, show how the Mean Absolute Deviation can be found using compute software. Open the MAD Fathom file. This file has been completed to find Mean Absolute Deviation. The teacher, however, would start the simulation from the beginning in class. To do so, generate 50 random integers between 1 and 100. Find the mean of the random numbers, use it to calculate the deviations, take the absolute value of the deviations, and then calculate the mean of the absolute deviations to find the Mean Absolute Deviation.

To finish class, the students will take the worksheet from Day 7/8 and find the Mean Absolute Deviation of each data set.